Quiz 1.

Soln 1. D: Shinchan has the disease
T: Shinchan tests positive for the disease

We know that

$P(D) = 0.01$ since the disease affects 1% of the population.

Also, $P(T|D) = 0.95$, $P(T^c|D^c) = 0.095$

We want to find P (Shinchan has the disease / Shinchan has tested positive for the disease) i.e. $P(D|T)$.

— 1 mark

We apply Bayes' rule.

$$P(D|T) = \frac{P(T|D) \cdot P(D)}{P(T)}$$ — 1 mark

The numerator is known. To find the denominator, we use the total probability law:

$$P(T) = P(T|D).P(D) + P(T|D^c) \cdot P(D^c)$$

— 1 mark

Thus, we have, $P(T|D) \cdot P(D) = 0.95 \times 0.01$

$P(T) = 0.95 \times 0.01 + 0.05 \times 0.99$

$\therefore P(D|T) = \frac{0.0095}{0.059} = 0.161$

— 2 marks.

We can conclude that even though the test is reliable, there is only a 16% chance that Shinchan has COVID-19 given that he tested positive.

Sol^n 2. a) The sample space for this problem can be written as:

$\Omega = \{BB, GG, GB, BG\}$ — 1 mark

B: Boy

G: Girl

We need to find $P(\text{Both children are girls} \mid \text{Elder child is a girl}) = \frac{P(\text{Both girls, Elder child is a girl})}{P(\text{Elder child is a girl})}$

(def^n of conditional prob.).

$\therefore$ $P(\text{Both are girls} \mid \text{Elder child is a girl})$

$$= \frac{1/4}{1/2} = \frac{1}{2}.$$ 2 marks.

b) The sample space Ω is the same for this problem as well.

The required prob is $P(\text{Both boys} \mid \text{Atleast one is boy})$

$$= \frac{P(\text{Both boys, atleast one is boy})}{P(\text{Atleast one is boy})}$$

$$= \frac{1/4}{3/4} = \frac{1}{3}.$$ 2 marks.

1 mark - identifying sample space for a) & b)
2 marks for each part done correctly.

Soln 3. Data : 40, 70, 50, 100, 90

a) Five figure summary :

Minimum : 40

(Q_1) First Quartile : 50 — 2 marks

Median : 70

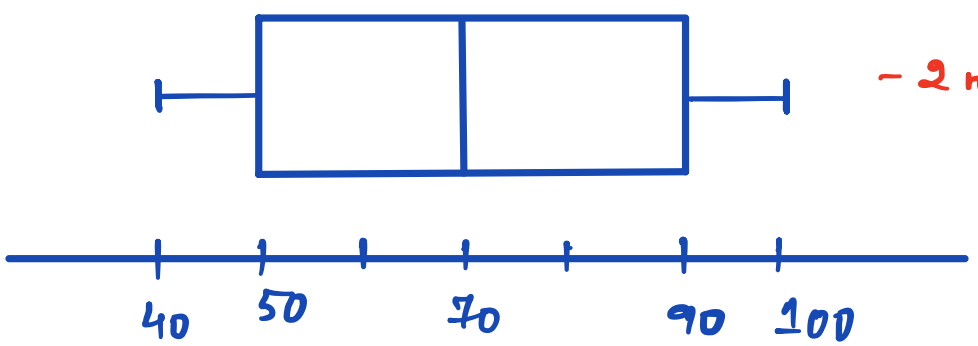
(Q_3) Third Quartile : 90

Maximum : 100

b) Interquartile range (IQR) $= Q_3 - Q_1 = 90 - 50 = 40$

— 1 mark

c) Boxplot



— 2 marks

d) All the points lie within 1.5IQR of the first and third quartiles, no point qualifies as an outlier. — 1 mark

e) Average = 70 — 1 mark

f) Standard deviation = 25.4951 — 2 marks.

g) Symmetric — 1 mark